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Abstract. In the last several decades, there has been an explosion of research concerning consciousness with some efforts at mathematical modelling. The purpose of this paper is to model conscious states using categorical constructions. In particular, this modelling captures temporality, the intentional structure of consciousness, and meaning fields, which provide the templates for occurrent events. The architecture of conscious states naturally lends itself to applications of category theory, more specifically to sheaf-like constructions in which the stacks over a site are topoi whose germs are the objects of the topoi. Each topos represents the potential experiential content of an individual where the objects represent meaning fields, the phenomenological subject is represented by the terminal object, and the morphisms from the terminal object into the objects, the elements, represent attentional targeting of the specific content of meaning fields yielding an individual's subjective experience. Temporality is represented by movement along the objects of the site, with different possible realities denoted by switching between contravariant functors over the site. This is an innovative way of modelling conscious states with the possibility of creating potentially useful ways of thinking about them that could then be empirically tested.

Mathematics Subject Classification (2010): 91E10

Keywords. Consciousness, mathematical modelling, category theory, topoi, sheaves.

The idea in this paper is to use category theoretical structures to model some of the distinctive features of conscious states. This follows upon earlier work that I have done using Grothendieck topoi to model conscious states as conceptualized by Edmund Husserl (Barušs, 1989) and the flicker aspect of physical reality in collapse-type quantum mind theories (Barušs, 2008). In that sense, the work presented here is a snapshot of a further stage of that modelling effort and not intended to be a completed model.

Why category theory? Category theory was invented by Saunders Mac Lane and Samuel Eilenberg as a way of classifying universal structures that recur in different areas of mathematics. If these structures are universal in mathematics, then it is possible that they occur in "reality" more generally, so that category theory could be an efficient way of conceptually capturing them. For instance, Mac Lane begins his discussion of categories by considering two transformations leading to the same result (Mac Lane, 1971). This is something that occurs in physical manifestation as well, so why not efficiently capture such sequences using categorical structures.

Category theory has arrived somewhat recently to mathematics, with the consequence that it has been under-utilized beyond the domain of its own investigation, and has not yet displaced older mathematical methods. For instance, the usual formalizations of quantum mechanics and quantum field theory depend upon the use of operators, Hilbert and Folk spaces, and so on, even though sheaves, which are also used in the modelling here, more naturally fit the nonlocal and contextual features of subatomic events than the tensor algebras (Abramsky & Brandenburger, 2011). Another beneficial aspect of categorical modelling in the context of quantum theory is its ability to model higher-level processes (Abramsky & Coecke, 2008). The use of categorical structures here has the same purpose of trying to capture the higher-level phenomenology of subjective states, rather than lower-level mechanisms, such as, for example, accumulator models do when used in decision theory. Indeed, Menas Kafatos and his colleagues have called for such applications of category theory and used them explicitly for modelling consciousness (Struppa, Kafatos, Roy, Kato, & Amoroso, 2002; Kafatos, & Kato, 2017). The modelling presented in this paper has been developed independently of their efforts.

There are different variations on category theory by different authors so that, to be clear, I am using the structures that arise in the context of elementary topoi, which are specific types of categories that embed internal logic, and, in particular, following the nomenclature used by Robert Goldblatt (1979), although it becomes necessary to create additional mathematical formalisms in the course of this modelling. I will alternate between giving heuristic descriptions of what I am doing that do not require any expert knowledge with technical information about the mathematical constructions and how I am using them for the modelling, which do require familiarity with basic categorical constructions.

1. Conscious States

The notion of "conscious states" refers to sequences of subjectively experienced events characterized by the presence of existential qualia, i.e., the subjectively felt sense that existence is going on (Barušs, 1987; Barušs & Mossbridge, 2017). Such events include both those that reference physical manifestation as well as those that are purely imaginary. There are several features of that broadly conceptualized domain that, in particular, are being modelled here.

1. Temporality. Based primarily on Julian Barbour's theory of time (Barbour, 2000) and my examination of observation in collapse-type quantum mind theories (Barušs, 2008), Julia Mossbridge and I developed a theory of temporality whose main idea is that occurrent events consist of an apparent sequence of *nows* that arise from a pre-physical substrate. Thus, there are two temporal streams: the usual apparent linear time stream, as experienced subjectively or objectively through the use of chronometers, and "deep time" which orders the sequence of experienced *nows*. In this conceptualization, each *now* has an associated "past" and "future," though these are not necessarily actual "pasts" and "futures" of deep time. Furthermore, there is a flicker aspect to reality, in that there is an "offset" between the "onset" of the *nows* (Barušs & Mossbridge, 2017).

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2. The view from somewhere. Conscious states are usually characterized by intentionality, in the sense that there exists a "self" directed toward "objects" of experience. While it is possible to infer a "view from nowhere," human experience is such that there is usually a view from somewhere (Nagel, 1986), arguably, even in cases of nondual consciousness.

3. Meaning fields. I developed the theory of meaning fields as a way of capturing what appears to be the presence of meaning beyond the human. For the purposes of this modelling, *meaning fields* can be regarded as patterns existing in reality that structure occurrent events (Barušs, 2018).

2. Categorical Structures

The categorical structures that are to be used for this modelling include categories, topoi, and sheaves. However, for the purposes of this modelling, I modify the sheaves and introduce the notion of partial functors. The following is a somewhat intuitive description of these structures, with proper precision introduced as needed. Usually composition of mathematical structures that can be associated, typically functions, are written using backward notation, e.g., $g(f(x))$, which means, first apply f to x and then apply g to $f(x)$. In this paper, for convenience, composition is written in forward notion, e.g., xfg, which means, first do x, then f, then g, with the exception of the membership relation, denoted by ε which is written in the usual, hybrid fashion.

A *category* is a collection of *objects* and *arrows* (also called *morphisms*), such that each arrow has an object as a *domain* and an object as a *codomain*. The arrows are closed under the operation of composition, they are associative, and identity arrows exist for each object. Perhaps the most obvious category is the category *Set* in which the objects are sets and the arrows are functions. The category of categories has categories as its objects, and *functors*, structure-preserving transformations, as its arrows. *Contravariant functors* change the directions of arrows in their image. A *topos* is a category with additional structure. Such structure includes a *terminal object* (usually denoted by *1*) which has the property that there is a unique arrow from every object in the topos to that terminal object.

Now we consider a contravariant functor, called a *stack*, from an arbitrary category C to *Set*. The initial category C is called a *base space*, the image of an object a is the object aF, called a *stack*, and an element x of aF is a *germ*. As we can see in Figure 1, germs are naturally carried forward from stack to stack.

What we notice here is that there is a lower layer of objects a, b, c and so on, joined by arrows f, g and so on. This is the base space. Only two objects and three arrows are shown here for illustrative purposes. There would be, in general, an infinite number of objects and arrows in a category. And there could be an infinite number of arrows between any two objects. Then there is an upper layer of balloons, which are just sets of elements. Again, there would be an infinite number of such balloons, one for each of the objects in the base space. The elements of those sets are called "germs." There are also arrows *fF* and *gF*, which were elevated by the functor *F* from the base space. Those arrows carry all of the germs from the stack *aF* to the stacks *bF* and *cF*. Now, we are going to introduce further structure, called a "pretopology," on the base space, that will allow us to add more features to the modelling.

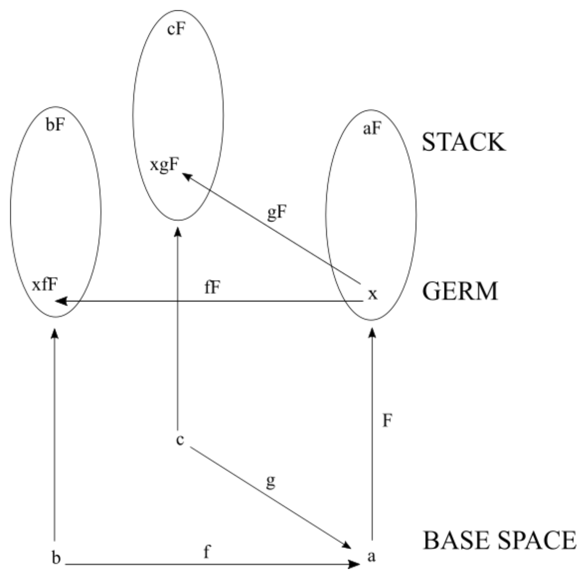

Figure 1

So, we further suppose that *C* has a *pretopology*, denoted by *Cov*, which is to say that there are *covers* of objects, which consist of collections of arrows with some additional properties, whose codomains are those objects. We also introduce a compatibility condition for the arrows of covers as follows: For a contravariant functor *F*, given any cover $\{f_x : a_x \rightarrow a : x \varepsilon X\} \varepsilon\, aCov$ and any collection of germs $\{s_x \varepsilon\, a_x F : x \varepsilon X\}$ that are pairwise compatible, i.e., that can be pulled back to the same germ in $(a_x \times_a a_y)F$, then there is exactly one $s_a \varepsilon\, aF$ so that $s_a\,(f_x F) = s_x,\ \forall x \varepsilon X$, where *X* is an index set and *Cov* is a pretopology. If a *compatibility condition* for germs on covers is satisfied then the stack *F* is a *sheaf*. The resultant construction is called a *Grothendieck topos*. We are actually going to only be using Grothendieck topoi as a stepping stone, so that, beyond naming them, we are not going to pursue the nature of such topoi.

3. Categorical Modelling

Now we pursue the modelling. The idea is to regard the base space as the temporal grid of *nows*, each of which is represented by an object, with the direction of the arrows indicating the future coming towards one at any given point in time. To clarify, the objects represent the onset of physical manifestation, the *nows*, and the arrows represent the transitions in deep time between *nows* that occur during the offset of physical manifestation.

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The stacks can be interpreted as the experiential worlds that occur at any given *now* for a person. However, for the purposes of this modelling, we need to enrich the structure of the stacks, so we replace the sets that make up the stacks with elementary topoi. Elementary topoi are categories that are generalizations of both Grothendieck topoi and sets, with the word “elementary” referring to the fact that they are defined axiomatically in first-order logic. This is an additional mathematical feature introduced for the purposes of this modelling that is not part of existing sheaf constructions. Now the objects of topoi become the germs and we let them represent the meaning fields. The relationships between the meaning fields are captured by the arrows between the objects which, in *Set*, would just default to subsets, intersections, and so on. The point of using topoi rather than just categories of sets in this modelling is to signify the notion that the meaning of meaning fields is not exhausted by their informational content, in that an object of a topos need not have any elements, as sets do, and if it does, then they need not define the totality of an object’s dynamics.

To represent the notion of a view from somewhere, we designate the terminal object in a topos in a stack as the subjective aspect of intentionality. The fact that there is a unique arrow from every object to the terminal object represents the idea that the meaning fields in a person’s psyche can influence the structure of her experience. In a topos there can also be arrows from the terminal object to other objects in a topos. Such arrows are called *elements* and represented by $\varepsilon_1, \varepsilon_2, \varepsilon_3$, and so on. The contents of experience are going to be represented by these elements, thereby modelling the use of explicit or implicit attention to extract specific contents from meaning fields. Note that the metaphorical arrow of intentionality is modelled as an actual collection of arrows from the terminal object to objects in a topos. I call that collection a *spray*. It is shown in Figure 2.

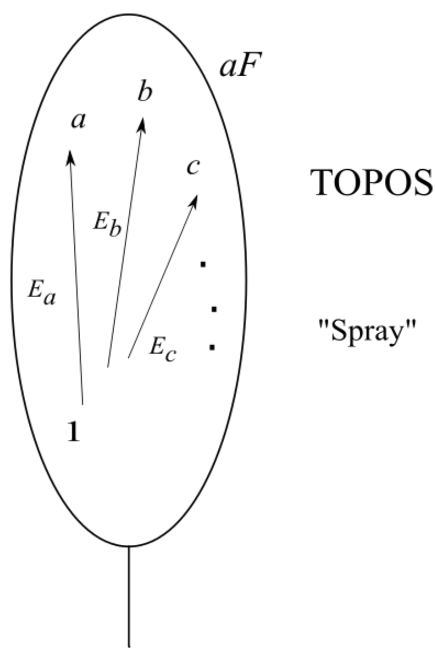

Figure 2

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Changes in a person's experience from *now* to *now* are represented by changes to the spray. In other words, some elements disappear whereas others are added in the transition from one *now* to the next. To do this mathematically, I introduce the notion of a *partial functor* as a functor that acts on all of the objects and arrows of a topos except for the arrows that are not being carried forward. In other words, a partial functor, by definition, is a functor that acts on a subcategory of a category. Furthermore, we suppose that there is a collection of such partial functors f_1F, f_2F, f_3F and so on, indexed by corresponding arrows f_1, f_2, f_3 and so on, in the base category, that allows for all possible permutations of sprays. This is represented in Figure 3.

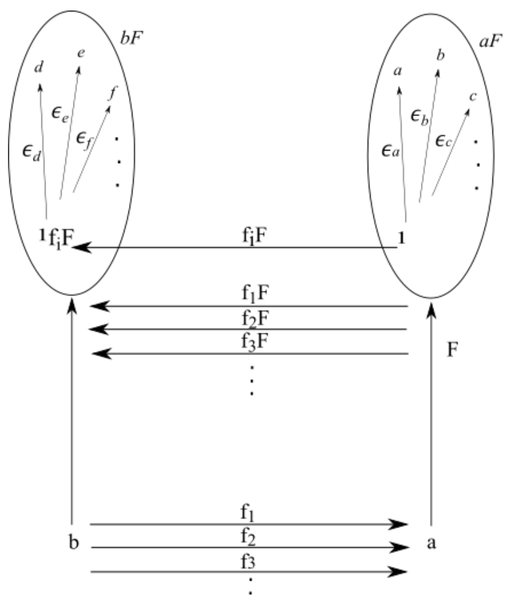

Figure 3

An individual could end up in different *nows* with possibly different additional meaning fields. This can occur by being carried to a different future *now* by an arrow in the base space, say, arrow *g* rather than arrow *f*, with the meaning fields corresponding to the domain of *g* rather than the domain of *f*. This is illustrated in Figure 4.

An individual could change realities more dramatically, as represented by shifting to a different contravariant functor at any given *now*, with the sprays being carried over to new ones by *natural transformations*, the morphisms that carry arrows to arrows across functors. This could represent shifting to a radically different reality with different pasts and futures. This becomes significant when considering how a person could dramatically alter reality from *now* to *now*. That is to say, there could be ways of using one's volition to not only switch between meaning fields but change to a reality with different meaning fields (cf. Barušs, van Lier, & Ali, 2014). In Figure 5, natural transformations τ and σ carry sprays to the topoi in the image of the contravariant functor *G*, which represents a radically different version of reality. To keep the diagram simple, only a single

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element from a spray is shown for each modified stack.

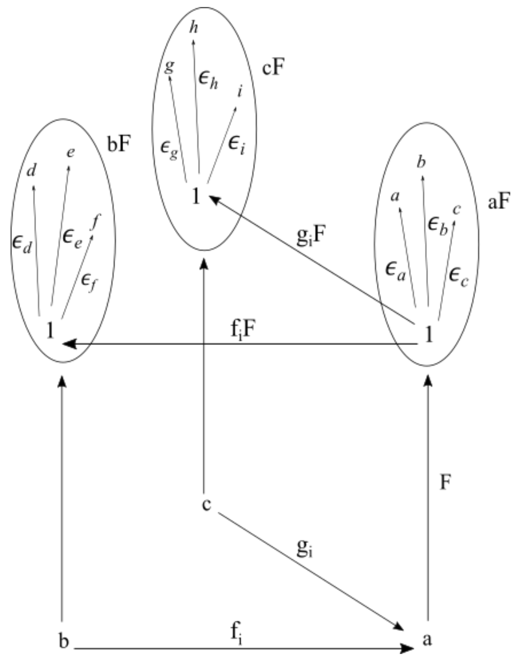

Figure 4

Temporally, with regard to the base space, we can think of an individual going down all possible pathways, consistent with multiverse theories and relational quantum mechanics, or think of her as choosing a single path, consistent with collapse-type quantum theories. This modelling does not distinguish between those two alternative interpretations of quantum theory.

4. Modelling Additional Features of Conscious States

We have modelled the three core features of conscious states that we set out to model — temporality, the view from somewhere, and meaning fields — in a somewhat natural way. But these topos-theoretic structures are richly endowed with features that could also be used in the modelling. This section presupposes working familiarity with basic categorical constructions.

Every topos carries *internal logic*, which could perhaps be fruitfully exploited. A basic example is that of assigning the value *true* whenever there is a monomorphism from a meaning field to another meaning field. In the topos *Set* this would mean that the value *true* is assigned whenever a meaning field is a subset of another meaning field. In particular, by the Ω-Axiom: for every monic $f{:}a \rightarrowtail d$ there is a unique arrow $\chi_f{:}d \to \Omega$ so that the diagram in Figure 6 is a pullback square. What this means is that the diagram takes the value *true* precisely when the meaning field represented by *a* is included in the meaning field represented by *d*. This means that an element of *a* is also an element of *d*, and there is sufficient logical structure inside the model to assign a truth value to that relationship. There could be additional ways of exploiting the rich internal logic of topoi for modelling conscious states.

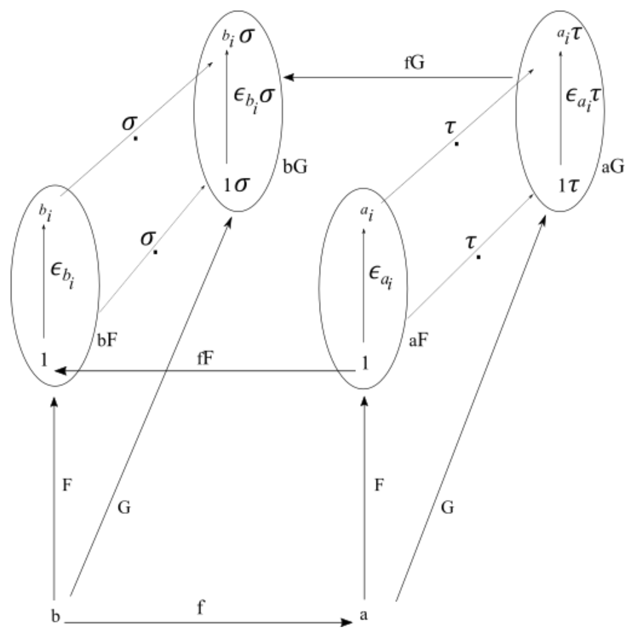

Figure 5

There is good evidence for the occurrence of presentiment, a person's anomalous ability to anticipate future events to which she has not been sensorially exposed (Bem, 2011; Barušs & Mossbridge, 2017). We recall the compatibility condition for sheaves and generalize it to our case in which the stacks made of sets have been replaced with stacks made of topoi. The idea is that if a meaning field exists in a suitable arrangement of future states, represented by a cover of an object in the base space, then that meaning field already exists in the stack over that object. In this way presentiment can be modelled.

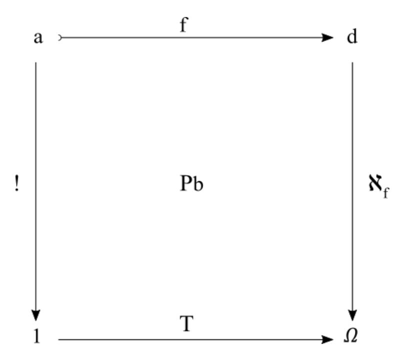

Figure 6

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There has been speculation that there is only a single subjective sense experienced as a multiplicity by individuals (Carse, 2005). We model such unity in diversity using the terminal object in the category of topoi, namely the *degenerate topos,* by considering a collection of functors from the degenerate topos to each of the topoi in the category that picks out the terminal object in each topos. In this way, all of the subjective senses are linked to a single subjective sense represented by the object in the degenerate topos. And we have modelled the idea there is only a single subjective self.

5. Conclusion

With this modelling, I have represented *nows* as the objects of a base category, with stacks as the domain of experiential events. Meaning fields are the objects of the topoi that constitute the stacks. Subjectively experienced events are represented by sprays of elements in a topos stack. Changes to experienced events are represented by partial functors between the stacks. Radical transformation of reality is represented by natural transformations of contravariant functors. Presentiment is represented by an application of the compatibility condition for sheaves. And the notion of a universal self is represented by functors from the degenerate topos. This is an outline of a mathematical model of conscious states that could potentially be further developed as a somewhat different way of conceptualizing cognitive architecture, including the utilization of the internal logic of topoi.

Note

This paper is based on a talk "Categorical Modelling of Meaning Fields" given by the author at the 51st Annual Meeting of the Society for Mathematical Psychology at the University of Wisconsin, Madison, July 21–24, 2018.

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